

01.02.2024 - I Prize Winner - Ms.Naga Bhavya Sree Solution

Solution:

As DE is the angular bisector,

$$\frac{CD}{AD} = \frac{CE}{AE} \text{ ----- (1)}$$

⇒ For BE & $\triangle ADC$,

$$\left| \frac{OA}{OD} \cdot \frac{DB}{BC} \cdot \frac{CE}{AE} \right| = 1$$

$$\left[\frac{AD-OD}{OD} \right] \cdot \left[\frac{BD}{BD+CD} \right] \cdot \frac{CD}{AD} = 1 \quad \text{[from (1)]}$$

$$\left[\frac{CD}{OD} - \frac{CD}{AD} \right] \cdot \left[\frac{BD}{BD+CD} \right] = 1$$

$$\frac{CD}{OD} - \frac{CD}{AD} = 1 + \frac{CD}{BD}$$

$$\frac{1}{OD} - \frac{1}{AD} = \frac{1}{CD} + \frac{1}{BD}$$

$$\Rightarrow \frac{1}{AD} + \frac{1}{BD} + \frac{1}{CD} = \frac{1}{OD} \text{ ----- Hence Proved}$$


